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Displacement Dependent Friction in Space Structural Joints

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Introduction

ONE of the less well understood aspects of the dynamics of structures is the origin and characteristic of their structural damping. At least three sources of passive energy dissipation exist in a space structure: material damping, passive damping elements,[1,2] and joints and fittings. Energy dissipation in joints takes place due to the relative motion of the contacting surfaces, which leads to both frictional and impact losses.

Ideally, the joints of a multielement structure are designed either to stiffen the ends of the structural members, as in a clamped end, or to allow the members one or more degrees of freedom, as in a pinned end. Realistically, motion occurs in the *rigid* joint since bending in the structural member causes the joint to deform elastically and motion is impeded in the *pinned* joint since friction occurs about the pin. In both cases, motion in the joint results in energy dissipation. This Note focuses on modeling the energy dissipation in two representative joint models. The model of friction used is one with a nonconstant coefficient. The frictional coefficient is zero at the mean displacement and varies linearly with the absolute magnitude of displacement.

Displacement Dependent Frictional Damping

One measure of damping useful in analyzing friction is the loss coefficient g related to the energy loss per cycle ΔE divided by the peak strain energy E

$$g = \frac{\Delta E}{2\pi E} = \frac{1}{2\pi E}\int F_f(\xi)\mathrm{d}\xi \qquad (1)$$

where $F_f(\xi)$ is the frictional force. In the case of Coulomb friction, the friction force is constant and proportional to the normal load. The energy loss per cycle varies linearly with amplitude and the total energy of the system varies with the square of the amplitude. Therefore, the loss coefficient varies inversely with displacement. If the load normal to the frictional surfaces in joints is due to gravity or a mechanical preload, the conventional model of constant normal load may be appropriate.

In the absence of gravity, the origin of the normal load in a space structure joint may be due to local or global deformation of the structure.[3] If the normal force varies linearly with the absolute value of displacement, the dependence of the friction force on displacement is

$$F_f = -\mu c\,|\xi|\,\mathrm{sign}(\dot{\xi}) \qquad (2)$$

where the force is assumed to be zero with no deflection. With this frictional model, the loss per cycle ΔE varies as the square of the amplitude of displacement. Thus, displacement dependent friction yields the same functional dependence on motion as the conventional hysteretic model of material damping. For this case, where the friction force varies linearly with displacement and there is no mean load on the joint, the loss coefficient is independent of amplitude, as in the case of the conventional model of material damping.

Beam/Sleeve Joint

One joint concept for use in an erectable space structure is the beam/sleeve joint. In this concept, a concentric outer sleeve is used to stiffen a beam in bending. Both the beam and sleeve are compliant and, as the beam deflects, a moment is exerted on the sleeve, locally deforming it. An idealized model of the beam/sleeve joint is shown in Fig. 1. The beam is assumed to be simply supported at $x=0$ and elastically restrained by the joint at $x=\ell$. As the beam deflects, the corner of the beam rotates and deforms the joint, which has an effective spring constant k_j. The energy dissipation occurs due to friction between the corner of the beam and the joint. For small displacements, the friction force is dependent on the displacement of the beam

$$F_f = -\mu k_j \ell_j\,|w'(\ell,t)|\,\mathrm{sign}[\dot{w}'(\ell,t)] \qquad (3)$$

The boundary conditions for the assumed Bernoulli-Euler beam equations are those for a simply supported beam with an elastic restoring moment applied at the joint ($x=\ell$),

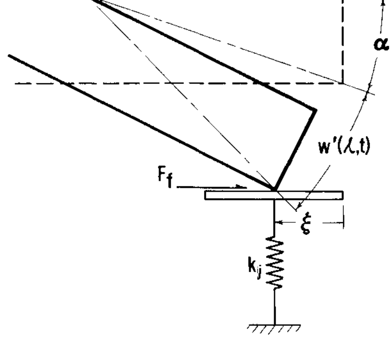

Fig. 1 Beam/sleeve joint model.

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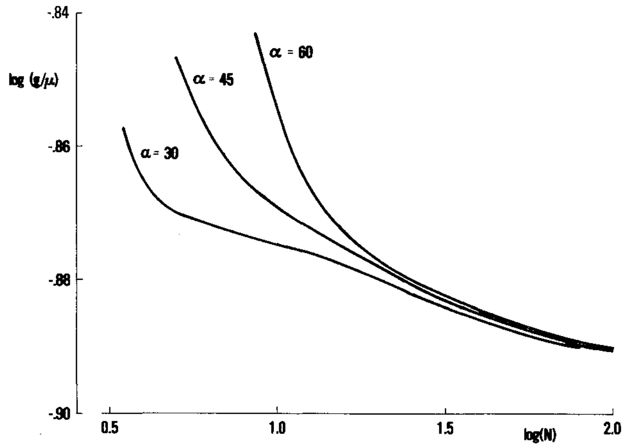

Fig. 2 Energy dissipation vs stiffness ratio for a beam/sleeve joint.

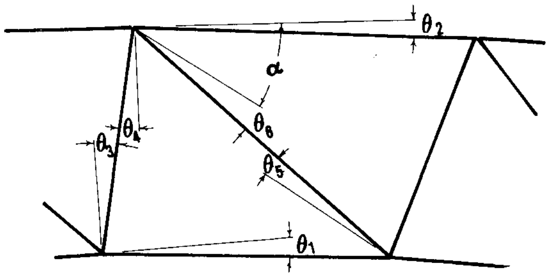

Fig. 3 A deformed bay of a two-dimensional truss structure.

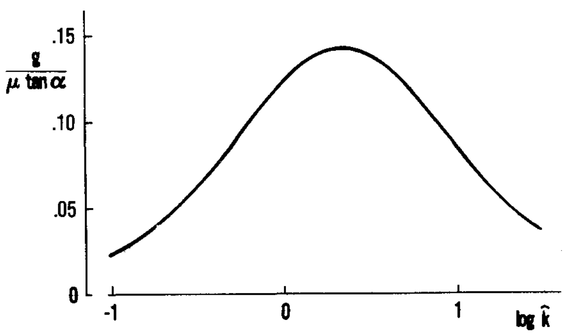

Fig. 4 Variation of loss coefficient with truss length ($r/\ell = 0.1$, $P/EA = 10^{-6}$).

$$w''(\ell,t) = \frac{-M}{EI} = \frac{-k_j}{EI}\ell_j^2 w'(\ell,t) \qquad (4)$$

where the segment of the beam in the joint has been assumed rigid. For sinusoidal motion of frequency Ω, the solution can be expanded in terms of the normal modes of the beam which are affected by the relative stiffness of the beam and joint. If it is assumed that the motion of the beam may be approximated by its first mode, the displacement of the corner of the beam in the joint is

$$\xi = \ell_j \tan\alpha A_1 \sin(\Omega t)\phi_1'(\ell) \qquad (5)$$

where A_1 is the amplitude of the first mode and α a small angle based on the geometry of the joint.

The loss coefficient is found by dividing the energy dissipated by the sum of the strain energies in the beam and joint,

$$g = \left[\frac{2}{\ell}\mu k_j \ell_j^2 \tan\alpha \phi_1'(\ell)\right] \Big/ \left[EI\int_0^\ell \phi_1''^2(x)\mathrm{d}x + k_j \ell_j^2 \phi_1'^2(\ell)\right] \qquad (6)$$

Substituting for the first mode, further manipulation results in the loss coefficient in the form

$$g = \mu\frac{8}{\pi}\left\{\tan\alpha \Big/ \left[\hat{k}\left(\frac{1}{\sin^2\lambda_1\ell} - \frac{1}{\sinh^2\lambda_1\ell}\right) + 1\right]\right\} \qquad (7)$$

where

$$\hat{k} = \frac{1}{2}\frac{k_j}{EI/\ell^3}\left(\frac{\ell_j}{\ell}\right)^2 \qquad (8)$$

is a nondimensional stiffness parameter representing the ratio of the rotational stiffness of the joint to the effective stiffness of the beam. The eigenvalue λ_1 resulting from the solution of the homogeneous beam equation is also a function of the stiffness parameter.

Damping in the beam/sleeve joint depends on the coefficient of friction, the geometry of the joint, and the relative stiffness of the beam and joint $\hat{k}$. The loss coefficient divided by $\mu\tan\alpha$ is plotted vs the stiffness ratio in Fig. 2. For either very rigid ($\hat{k} \to \infty$) or very flexible ($\hat{k} \to 0$) sleeves, there is no energy dissipation. The rigid sleeve cantilevers the beam, preventing motion. The flexible sleeve does not exert a restoring normal force. For realistic joints, where both relative motion and an elastic restoring force occur, there is finite dissipation. It follows that there exists a relative stiffness ratio for maximum energy dissipation. As can be seen in Fig. 2, the maximum energy dissipation results when the stiffness parameter $\hat{k}$ is approximately two, which indicates that the resistance to applied moment of the joint should be of the same order as that of the connecting beam.

Truss/Pin Joint

A truss/pin joint is designed to have a rotational degree of freedom about the pin. The truss member experiences an axial load that serves as a displacement dependent normal load creating a frictional force on the pin. As the structure deforms, energy is dissipated due to friction between the pin and socket. The nth bay of a typical two-dimensional truss structure is shown in Fig. 3. Each bay of the truss has four pin joints about which the four bars rotate. For the six relative angles of rotation in the bay θ_j, there are six contributions to the energy dissipation

$$\Delta E_n = \sum_{j=1}^{6} 2\mu r (P_j\theta_j)_n \qquad (9)$$

where r is the radius of the pin and P_j the axial load. Similarly, the peak strain energy in the nth bay is the sum of the strain energies of each member. For a truss of N bays, the loss coefficient is

$$g = \frac{2\mu}{\pi}\frac{r}{\ell}f(N,\alpha) \qquad (10)$$

where $f(N,\alpha)$ represents a nondimensional function dependent only on the geometry of the truss

$$f(N,\alpha) = \left[\sum_{n=1}^{N}\sum_{j=1}^{6}\frac{P_j(N,\alpha)}{P}\frac{EA_0}{P}\theta_j(N,\alpha)\right] \div \left[\sum_{n=1}^{N}\sum_{i=1}^{6}\frac{P_i(N,\alpha)^2}{P^2}\frac{EA_0}{EA_i}\right] \qquad (11)$$

The loss coefficient vs total number of bays is plotted in Fig. 4 for three values of diagonal brace angle and for $r/\ell = 0.1$ and $P/EA = 10^{-6}$. Ashley[4] examined the effect of structural length on damping and estimated that damping would fall off as the structural length to powers between 0 and -1. For the three trusses examined, damping decreases with length to powers of -0.1 for the short truss with diagonal angles of

60 deg to approximately -0.01 for the long trusses. The variation is explained by considering the two contributions to beam deflection, bending, and shear. Equivalent beam bending in a truss is due primarily to compression and tension of the truss members, while equivalent shear is due primarily to the relative motion of the adjacent bays. Since tension and compression result in little relative motion of the bays, the equivalent shear is the primary contributor to energy dissipation. Just as shear deflection becomes a small percentage of the total deflection as a beam is lengthened, the level of damping decreases with truss length.

Conclusions

Displacement dependent friction can be used to model the damping in several common types of truss joints in which the frictional forces are due to elastic deflection rather than mechanical preloads. In sleeve-stiffened beam joints, a maximum friction damping is obtained when the relative rotational stiffness of the joint and beam are of the same order. For pin joints in multielement trusses, a maximum frictional damping occurs for trusses of low length/bay-depth ratio, and large pin-radius/bay-depth ratio. The losses increase as the coefficient of friction is increased.

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Bending Eigenfrequencies of a Two-Bar Frame Including the Effect of Axial Inertia

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Introduction

When considering free lateral vibrations of slender bars and framed structures, the axial force due to the longitudinal inertia is often disregarded within the scope of the linear dynamic analysis. Hohenemser and Prager[1,2] have investigated the influence of the longitudinal inertia upon the natural frequencies of free lateral vibrations. They have shown that the foregoing simplification is reasonable for frames consisting of bars with slenderness ratios greater than 40; for bars with smaller slenderness ratios, the omission of this effect may lead to errors as high as 10%.

This paper reconsiders the aforementioned effect in case of the free lateral vibrations of frames, including the translational inertia of the mass of their joints. To this end, a two-bar frame with various bar slenderness ratios, moments of inertia, and length ratios is considered. To the knowledge of the author, the effect of the longitudinal inertia of a concentrated mass upon the flexural eigenfrequencies has been discussed only for the case of dynamic buckling of a simply supported beam.[3]

Formulation of the Problem

We consider the free lateral motion of the uniform two-bar frame shown in Fig. 1, supported on two immovable hinges, whose joint mass is equal to M. Each bar is of length ℓ_i, mass per unit length m_i, cross-sectional area A_i, and moment of inertia $I_i\,(i=1,2)$. Let $w_i(x)$ and $\xi_i(x)$ be the lateral and axial displacement components referring to the centerline of the ith bar.

The differential equations governing the motion of the frame are

$$\begin{aligned} EI_i w_i''''(x_i,t) + m_i \ddot{w}_i(x_i,t) &= 0 \\ EA\xi_i''(x_i,t) - m_i \ddot{\xi}_i(x_i,t) &= 0 \end{aligned} \qquad (i=1,2) \qquad (1)$$

The associated boundary conditions are

$$w_i(0,t)=0, \quad \xi_i(0,t)=0 \qquad (i=1,2)$$
$$w_1'(\ell_1,t) = w_2'(\ell_2,t)$$
$$w_1(\ell_1,t) = \xi_2(\ell_2,t), \quad w_2(\ell_2,t) = -\xi_1(\ell_1,t)$$
$$w_i''(0,t)=0 \qquad (i=1,2)$$
$$EI_1 w_1'''(\ell_1,t) - EA_2\xi_2'(\ell_2,t) - M\ddot{w}_1(\ell_1,t) = 0$$
$$EI_2 w_2'''(\ell_2,t) + EA_1\xi_1'(\ell_1,t) - M\ddot{w}_2(\ell_2,t) = 0$$
$$EI_1 w_1''(\ell_1,t) + EI_2 w_2''(\ell_2,t) = 0 \qquad (2)$$

For a free motion, one may assume

$$w_i(x_i,t) = W_i(x_i)e^{i\omega t}, \quad \xi_i(x_i,t) = \Xi_i(x_i)e^{i\omega t} \qquad (3)$$

where $i=\sqrt{-1}$ and ω is the circular frequency of the motion.

To facilitate the solution, we introduce the following dimensionless quantities:

$$\bar{x}_i = x_i/\ell_i, \quad \bar{w}_i = W_i/\ell_i, \quad \bar{\xi}_i = \Xi_i/\ell_i$$
$$k_i^4 = m_i\ell_i^4\omega^2/EI_i, \quad \lambda_i^2 = A_i\ell_i^2/I_i, \quad \upsilon_i^4 = k_i^4/\lambda_i^2$$
$$\mu = I_2/I_1, \quad \rho = \ell_2/\ell_1, \quad \bar{M} = M/m_1\ell_1 \quad (i=1,2) \qquad (4)$$

By means of Eq. (4), Eqs. (1) and (2) become

$$\bar{w}_i'''' - k_i^4\bar{w}_i = 0 \text{ and } \bar{\xi}_i'' + \upsilon_i^4\bar{\xi} = 0 \quad (i=1,2) \qquad (5)$$

and

$$\bar{w}_i(0)=0, \quad \bar{\xi}_i(0)=0 \qquad (i=1,2)$$
$$\bar{w}_1'(1) = \bar{w}_2'(1), \quad \bar{w}_1(1) = \rho\bar{\xi}_2(1), \quad \bar{w}_2(1) = -(1/\rho)\bar{\xi}_1(1)$$
$$\bar{w}_i''(0) = 0 \qquad (i=1,2)$$
$$\bar{w}_1'''(1) - (\mu/\rho^2)\lambda_2^2\bar{\xi}_2(1) + \bar{M}k_1^4\bar{w}_1(1) = 0$$
$$\bar{w}_2'''(1) + (\rho^2/\mu)\lambda_1^2\bar{\xi}_1'(1) + (\bar{M}\rho^3/\mu)k_1^4\bar{w}_2(1) = 0$$
$$\rho\bar{w}_1''(1) + \mu\bar{w}_2''(1) = 0 \qquad (6)$$

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